

# XV EITA RESEARCH MEETING IN APPROXIMATION THEORY 2020

Organizado por el Grupo "Análisis Matemático y Aplicaciones" (UZ)

Zaragoza (on line), 5-6 de noviembre de 2020

<http://eventos.unizar.es/go/xveita2020>



## PONENTES

5 de noviembre (12-13:30)

Jesús Oliva-Maza (UZ)

Javier González (ICMAT)

Oscar Blasco (UV)

6 de noviembre (12-13:30)

Miguel Monsalve (UCM)

Renato Álvarez (US)

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## Lie group representations on operator ranges

J. Oliva-Maza,

Let  $\mathcal{R}$  be an operator range of a Hilbert space  $\mathcal{H}$ , let  $\Lambda$  be the algebra of bounded operators on  $\mathcal{H}$  leaving  $\mathcal{R}$  invariant, and let  $\rho : G \rightarrow \text{GL}(\mathcal{H})$  be a Lie group representation. We are interested in those elements in the range of  $\rho$  that are in  $\Lambda$ , which is a relatively unexplored topic in the literature. Our motivation is to gain some insight in the representations of operators acting on a family of operator ranges of the Hardy space.

In one direction, we consider the case where the whole range  $\rho(G)$  is  $\mathcal{R}$ -invariant. For such a representation  $\rho$  there always exists a natural associated representation  $\tilde{\rho} : G \rightarrow \mathcal{B}(\mathcal{R})$ , for a suitable topology in  $\mathcal{R}$ . We characterize the continuity and smoothness of  $\tilde{\rho}$  in terms of the tangent map  $d\rho$ . On another direction, we give some basic properties about the algebra  $\mathcal{U} \cap \Lambda$ , where  $\mathcal{U}$  denotes the set of unitary operators on  $\mathcal{H}$ . These are results obtained during a (very short) PhD stay supervised by D. Beltita.

**Keywords:** Operator ranges, Lie group representations.

**Mathematics Subject Classification 2010:** Primary 46C07, 47A15; Secondary 22E66

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## Invariant Subspaces For Positive Operators on Banach Spaces With Unconditional Basis

F. Javier González-Doña (ICMAT- UCM)

We consider the invariant subspace problem for positive operators on Banach spaces with unconditional basis. We show that the Theorem of Abramovich, Aliprantis and Burkinshaw ([1]) does not characterize the existence of invariant subspaces (moreover, invariant ideals) for these operators. Motivated by this result, we consider lattice homomorphisms on Banach lattices whose order is induced by an unconditional basis. We characterize the matrix representation of these operators and use it to show that every lattice homomorphism on these spaces have non-trivial invariant subspaces. Indeed, we show they have non-trivial invariant ideals. Finally, we extend a result of Grivaux ([3]) on the existence of invariant subspaces for positive tridiagonal operators. This is part of a joint work with Eva Gallardo-Gutiérrez and Pedro Tradacete ([2]).

**Keywords:** Banach lattices, lattice homomorphisms, invariant subspaces

**Mathematics Subject Classification 2010:** 46A40, 46B40, 47B60

### References

- [1] Y. A. Abramovich, C. D. Aliprantis, and O. Burkinshaw, *Invariant subspaces of operators on  $\ell^p$ -spaces*, J. Funct. Anal. **115** (1993), no. 2, 418–424.
- [2] E. A. Gallardo-Gutiérrez, F.J. González Doña, P. Tradacete, *Invariant subspaces for positive operators on Banach spaces with unconditional basis*, <https://arxiv.org/abs/2005.01150>, (2020).
- [3] S. Grivaux, *Invariant subspaces for tridiagonal operators*, Bull. Sci. Math. **126** (2002), no. 8, 681–691.

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## Bilinear multipliers on Banach function spaces

Oscar Blasco,

Let  $X_1, X_2, X_3$  be Banach spaces of measurable functions in  $L^0(\mathbb{R})$  and let  $m(\xi, \eta)$  be a locally integrable function in  $\mathbb{R}^2$ . We say that  $m \in \mathcal{BM}_{(X_1, X_2, X_3)}(\mathbb{R})$  if

$$B_m(f, g)(x) = \int_{\mathbb{R}} \int_{\mathbb{R}} \hat{f}(\xi) \hat{g}(\eta) m(\xi, \eta) e^{2\pi i \langle \xi + \eta, x \rangle} d\xi d\eta,$$

defined for  $f$  and  $g$  with compactly supported Fourier transform, extends to a bounded bilinear operator from  $X_1 \times X_2$  to  $X_3$ .

In this talk we present some properties of the class  $\mathcal{BM}_{(X_1, X_2, X_3)}(\mathbb{R})$  for general spaces which are invariant under translation, modulation and dilation, analyzing also the particular case of r.i. Banach function spaces. We shall give some examples in this class and some procedures to generate new bilinear multipliers. We shall focus in the case  $m(\xi, \eta) = M(\xi - \eta)$  and find conditions for these classes to contain non zero multipliers in terms of the Boyd indices for the spaces.

**Keywords:** bilinear multipliers, Banach function spaces, rearrangement invariant spaces

## References

- [1] BLASCO, O. Notes on the spaces of bilinear multipliers. *Rev. Unin Mat. Argentina* **50**(2), 20–34, 2009.
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- [3] BLASCO, O. Bilinear multipliers on Banach function spaces. *J. Funct. Spaces* **7639380**, 11p, 2019.

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## Bishop operators: invariant subspaces and spectral theory

Miguel Monsalve López,

Bishop operators were proposed in the fifties as candidates of Banach space operators having no non-trivial closed invariant subspaces. Despite many efforts and partial results, the existence of invariant subspaces for arbitrary Bishop operators is still an unsolved question.

Our objective in this talk is to depict a thorough overview of some recent advances concerning the existence of invariant subspaces for Bishop operators: mainly focusing on certain spectral properties which might be playing a crucial role in this problem.

The bulk of results presented along this talk have been developed as part of author's PhD thesis [5], done under supervision of Prof. Eva A. Gallardo-Gutiérrez.

**Keywords:** Operator Theory, Spectral Theory, Banach spaces and invariant subspaces

**Mathematics Subject Classification 2010:** 47A15, 47B37, 47B38

## References

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- [2] E. A. GALLARDO-GUTIÉRREZ AND M. MONSALVE-LÓPEZ. Power-regular Bishop operators and spectral decompositions. *Journal of Operator Theory* (in press).
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- [5] M. MONSALVE-LÓPEZ. *Bishop operators: invariant subspaces and spectral theory*. PhD-Thesis, Universidad Complutense de Madrid, 2020.

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## El covid-19 en números

R. Alvarez-Nodarse,

A principios de este 2020 apareció en escena un nuevo patógeno: el SARS-CoV-2 causante de la covid-19, enfermedad que comenzó siendo una especie de “gripe” pero que se ha convertido en una pandemia mundial. Desde marzo de 2020 la comunidad científica se ha volcado en la investigación de dicha enfermedad y como no podía ser menos, las matemáticas han aportado (o al menos lo han intentado) su granito de arena. En esta charla de carácter muy general vamos a discutir varios “números covid-19”. La motivación original era intentar dar una base rigurosa a afirmaciones como “lávate las manos y no te toques la cara”, mantén una “distancia de seguridad de ... metros”, o aportar soluciones al problema de cómo colocar a los alumnos en las clases para que estén lo más lejos posible. La mayoría de los resultados que se presentarán han sido publicados en el blog del IMUS [1] a lo largo del tiempo que llevamos lidiando con la covid-19 y han sido en colaboración con los profesores Francisco J. Esteban de la Universidad de Jaén y Niurka R. Quintero de la Universidad de Sevilla.

## References

[1] Blog del IMUS. <https://institucional.us.es/blogimus/categoria/emergencia-covid-1/>

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